# **Conventional Estimation Techniques**

## Preliminary Considerations

### Search Neighbourhood

#### Number of samples

A minimum of 4 is required to get an estimate and 12 samples is reasonable to get an estimate. Adding more points beyond the maximum 32 will not significantly improve the estimate.

#### Size of Neighbourhood

#### Shape of Neighbourhood

### Cross-validation

## Linear Kriging Procedures

The estimated value of the variable is linearly related to nearby samples.

### Simple Kriging

Value at the unsampled location can be estimated as: (similar to [**MVUE**](#_Inference_of_Parameters))

Real value: Estimated value:

Although the above relations are written as in random variable, but in actual they are the actual samples at the mentioned location.

#### Applying Unbiased condition

under the condition of 1st order stationarity

#### Applying Minimum Variance Condition

**Variance** can only be found for random variables. and being the random variables & being the constant.

To minimize variance, we need to equate partial derivative of wrt .

In matrix form all the n equations can be written as:

Covariance values from the model variogram calculated earlier.

Value of will be replaced in the above **equation 1** to get the value of

#### Minimum variance relationship in terms of variogram/semivariance

From :

In matrix form all the n equations can be written as:

is required to calculate the values.

#### Calculating error variance

Multiplying with and summing over all the values of :

Replacing in to get as below:

has the maximum impact to calculate the error variance.

in terms of the variogram/semivariance values:

The maximum value of error variance is the data variance . The error variance gets reduced with the weights assigned and the covariance amongst the sample points.

#### Characteristics of Simple Kriging

* + - * + Search Neighbourhood: It specifies which data are to be included in the weighted moving average. **Thumb rule:** Search radius ≥ Range from variogram (in the same direction). If using a uniform search radius in all directions, use the search radius comparable to the range of the long axis of the variogram.
        + In Kriging, clustered samples being similar to each other are assigned smaller weights as compared to an isolated point and away from other sample points. Thus, calculated Kriged weights always declusters the data.
        + Weights assigned are dependent upon two factors:

Stronger the relationship, larger is the assigned weight

Sample points spatial relationship to other sample points. Stronger the relationship to surrounding points, the less information that point can individually provide.

* + - * + If we do ‘normal score transform’ to the data, the variance/sill is always going to be 1.

### Simple Block Kriging

Similar to point kriging, we start with the equation:

#### Unbiased condition:

#### Applying Minimum Variance Condition

Similar to simple kriging to minimize variance, we need to equate partial derivative of wrt .

#### Calculating error variance

Notice the similarity between point and block kriging expressions except for covariance values either within a block or between blocks.

#### Characteristics of simple block kriging:

* + - * + It is more efficient to use block kriging than to use point kriging to estimate point values and then estimate the arithmetic average.
        + Block Kriging is useful for estimating block values of static properties that can be upscaled using arithmetic average. It is not useful for dynamic properties that are flow dependent like permeability.

### Ordinary Kriging

Similar to the Simple Kriging

Real value: Estimated value:

Although the above relations are written as in random variable, but in actual they are the actual values of the samples recorded at the mentioned location.

#### Applying Unbiased condition

Because we don’t know the value of the local mean, we can force to 0, which results in:

This is the *unbiased condition*. By forcing , we eliminate the requirement to know the mean value.

#### Applying Minimum Variance Condition

We must minimize the variance by taking into account the constraint defined in . To achieve this we adopt the Lagrange multiplier method and define a function as below:

To minimize variance, we need to equate partial derivative of wrt and .

In matrix form all the n equations from and can be written as:

Covariance values and are calculated from the model variogram.

From , we are able to calculate and thus

#### Calculating error variance

Multiplying with and summing over all the values of :

Replacing in to get as below:

#### Characteristics of Ordinary Kriging:

* + - * + In practice, the true global mean is rarely known, unless we assume the sample mean the same as the global mean. It has also been seen that the local mean within the search neighbourhood may vary over the region of interest. As a result the assumption of the first order stationarity can’t be strictly valid.
        + By forcing , we eliminate the requirement to know the mean value.
        + From calculations, SK is slightly better than OK for values smaller than the mean, but OK is better for value larger the mean. Also OK is more robust than the SK, since it nullifies the local mean each time it calculates and work with the residuals.
        + The variance estimation error is independent of the sample values.

##### Field Example: Ordinary Kriging Cross-validation

##### Field Example: Ordinary Kriging to generate Gross Thickness Maps

##### Field Example: Ordinary kriging to estimate original oil in place

##### Field Example: Block Estimation

### Cokriging

When simple kriging is extended to 2 variables. We assume they are *principal variable* and the *covariable*. The estimation equation can be written as:

Estimated value:

#### Applying Unbiased Condition

For *simple cokriging system*, is the equation of unbiased state.

**Option 1:** If the means and are not known and we force to 0, will result in an *ordinary cokriging system* as below.

& Thus,

But one of the drawbacks is some of the will result in negative weights, thus resulting in negative estimate at some locations which doesn’t have any physical meaning.

**Option 2:** Hence, to avoid the negative weighing, we’ve another option as below, where :

&

In the above relation, requires the knowledge of the means and

Thus the unbiased relations are as below:

**Option 1:** Replacing in ,

**Option 2:** Replacing in ,:

Depending, on which constraint is chosen or , we either proceed with or

#### Characteristics of Cokriging

* + - * + The limitations of cokriging:

Cokriging technique presumes a linear relationship between the variable and the covariables.

The relationship between the variable and the covariable must be strong. The strength of the relationship can be measures as follows:

The number of sample points used and the correlation coefficient. Higher the number of samples as well as the correlation coefficient, better is the relation.

Check if the relation is based on some physical factors. E,g, TWT is related to depth of the formation.

Historical evidence is examined like has any study like this has been conducted so far or has it been successful in the past.

If there is a weak relationship, covariable and cokriging must be used with caution.

Application of cokriging requires substantial spatial modelling effort.

### Cokriging: Ordinary Cokriging (Option 1)

**Condition**: & Thus,

**Variable**:

#### Calculating Minimum variance (Ordinary Kriging\_1)

Using constraint and thus the relation:

The error variance equation must be minimized alongwith the constraints from:

Taking derivatives of F wrt to and and equating to 0 we obtain:

The above set of equations can be written as in the matrix format below:

Where will give the values of and . **will have some negative values.**

#### Calculating Error variance (Ordinary Kriging\_1)

### Cokriging: Ordinary Cokriging (Option 2)

**Condition**: &

**Variable**:

Where the means , are known

#### Calculating Minimum variance (Ordinary Kriging\_2)

By using constraint and thus the relation:

is unchanged in all types of kriging: simple, ordinary kriging, because is not a *random variable* but a *constant* to variance.

The above set of equations can be written as in the matrix format below:

Where will give the values of and .

#### Calculating Error variance (Ordinary Kriging\_2)

=

**For option 2, we must know the means of the variables X and Y.**

Similar set of formulae can be generated for a **simple cokriging algorithm**.

### Cokriging: Simple Cokriging

Estimated value:

#### From Unbiased Condition

Using the unbiased condition, where we can calculate when are calculated and are known already (condition for Simple Kriging).

#### Calculating Minimum variance (Simple Kriging)

The above set of equations can be written as in the matrix format below:

Where will give the values of and . And thus

#### Calculating Error variance

=

### Collocated Cokriging

In this it is assumed, that to estimate , we must use the covariable value .

#### Applying Unbiased Condition

For *ordinary kriging*, we use the condition . Hence,

Where are assumed to be known and .

Thus,

#### Applying Minimum Variance

The error variance equation must be minimized alongwith the constraints from:

Taking derivatives of F wrt to and and equating to 0 we obtain:

The above set of equations can be written as in the matrix format below:

Where will give the values of s and . And can thus be resolved, when and are known.

The above matrix is dimensions. Further, we only need the variance of the variable Y, and no spatial model of Y (or variogram) is required. We need to calculate the cross-covariance () model to solve the above matrix. But as we know from Markov-Bayes approximation for cross-correlation:

.

So once we have modelled the variogram for the primary variable , we can easily calculate the cross-covariance of X and Y at the same lag distance, if we have already calculated it for 0 lag distance . So no independent modelling required for the cross-covariance.

### Generalized cokriging procedure

The principal variable is estimated using covariables.

Estimated value:

Where

#### Unbiasedness

From unbiasedness condition,

One of the options to satisfy the above relation would be:

and .

Other conditions can also be possible. Here the above relation is taken forward.

#### Minimum Variance

For minimum variance condition, and

,

#### Field Example: Cokriging to estimate permeability using Initial Potential

### Universal Kriging

It is applicable when estimation is required in presence of a trend and when we have a trend a first order stationarity is not satisfied. The local mean varies in the direction of the trend and is not reasonably close to the global mean.

Option 1: The variable of interest at the location

The drift which is also the mean

The drift-free residual satisfying the 1st order stationarity

The residuals , after the local trend removed, don’t exhibit any trend and mean of the residuals=0, satisfying the 1st order stationarity.

In principle, by applying the kriging technique to the residuals and adding back the trend we can estimate the values at the unsampled locations. Several options are available to estimate & model the variogram:

Determining the local means (drift) by using moving neighbourhood averaging method, subtracting the drift from the sample values and variograms of residuals are estimated.

The variogram is calculated in the direction of no-trend (satisfying 1st order stationarity) and applied in the direction of the trend.

Value at the unsampled location can be estimated as,

Trend in the data can be defined as,

In practice, a trend is rarely higher than the second order.

e.g. So,

#### Unbiased condition

Replacing in . Trend equation remains same throughout the area of interest.

To satisfy the above relation eq 5, we can impose the below condition,

for

The constraints are to be used for unbiased condition.

#### Minimum variance

for

for

for

for

In matrix format,

On solving the above matrix, we get the value of and .

#### Error variance

Summing all value of ,

Replacing, in the below equation

#### Characteristics of Universal Kriging

* + - * + Ordinary Kriging is a special case of the Universal Kriging esp. where L=0 only, the first.
        + We need to have prior knowledge about the trend, otherwise we wont be able to apply the universal kriging. Two options are used to circumvent this problem:

Use of intrinsic random functions of order k: This method assumes a more **generalized covariance model** of order k. The order k is determined by the order of the trend. If k=0 (no trend) the covariance model reduces to a variogram model. A procedure to similar to kriging is applied with the generalized covariance model.

Method of **external drift**: In this method, we assume that the trend in the data is provided by the secondary variable. The advantage is we don’t have to assume any specific trend for the primary variable, but the covariable has to be available at all the unsampled locations, like seismic data and the relationship between the drift and the secondary variable must be linear.

For example, the seismic data can provide information about the trend in the top of the interval.

where is the secondary variable value at the unsampled location.

### Kriging with external drift

It is an extension to the universal kriging, where the trend is assumed to be controlled by a secondary variable .

Value at unsampled location:

#### Unbiasedness

The above equation can be satisfied by considering the below constraints:

and

#### Minimum variance

Putting all the above equations in a matrix format

Thus we solve the value of , and .

#### Error variance

Summing all value of ,

since and

Replacing, in the below equation

One of the advantages of this, is it doesn’t require cross-covariance modelling.

#### Field Example: Universal Kriging to Estimate Top of reservoir

#### Field Example: Universal Kriging with an external drift to estimate permeability using porosity trend

### Generalized Covariance Function

Universal Kriging requires the knowledge of the following:

1. Covariance model of the residuals
2. Type of trend present in the original dataset

Instead of using a variogram as a spatial function, Matheron & Delfiner proposed to use a more generalized random function of order k. The principle behind it is a trend in the data can be removed by proposing an appropriate function and variogram is a special case of this generalized function.

The variance of the linear combination of the with weight value can be expressed in terms of generalized covariance function of order k.

Similar to universal kriging,

#### Unbiased condition of error

Replacing in . Trend equation remains same throughout the area of interest.

To satisfy the above relation eq 5, we can impose the below condition,

for

⇒ where

⇒ for

The constraints are to be used for unbiased condition.

Defining variable which is also the ***error***,

⇒

For example, For ,

For ,

For ,

If the data has no trends,

Then, where

And,

This is a *conventional variogram* multiplied by 2.

If the data has a linear trend,

Then, where L=1

And,

But,

since

Thus,

#### The generalized covariance function of order k:

Where where is the order of covariance.

E.g. for a linear model,

Whichever trend that minimizes, the difference between experimental and model is accepted.

Once covariance is modelled, equations similar to universal kriging can be used:

#### Error variance:

The advantage of this is:

1. Residual covariance need not be modelled
2. By determining the appropriate trend as part of the process, the type of trend does not need to be known.

### Kriging Summary

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | **Condition/**  **Requirement** | **Equation** | **Error Variance** |
| Simple Kriging |  |  | * Variogram-**X**: **Yes** |  |  |
| Simple Block Kriging |  |  | * Variogram-**X**: **Yes** |  |  |
| Ordinary Kriging |  |  | * Variogram-**X**: **Yes** |  |  |
| Ordinary Cokriging (Option I)  *Negative weights of covariable* |  | * And | * Variogram-**X/Y**: **Yes** * Covariogram-**XY**: **Yes** |  |  |
| Ordinary Cokriging (Option II)  *No negative weights of covariable* |  |  | * Variogram-**X/Y**: **Yes** * Covariogram-**XY**: **Yes** |  |  |
| Simple Cokriging |  |  | * Variogram-**X/Y**: **Yes** * Covariogram-**XY**: **Yes** |  |  |
| Collocated Cokriging  *Covariable from the same unsampled location* |  |  | * Vario.-**X/Y**: **Yes/No** * Covariogram-**XY**: **No** |  |  |
| Universal Cokriging  *With trends* |  |  |  |  |  |
| Kriging with external Drift  *Trend from covariable* |  |  |  |  |  |

## Non-linear Kriging techniques

### Log-Normal Kriging

**Steps for log-normal kriging:**

1. The original variable is subjected to log-normal estimation

1. Spatial relationship of the log-normally distributed variable is established
2. The value at the unsampled location is estimated using either simple or ordinary kriging. Error variance for variable Y is also calculated
3. The variable Y is back-transformed, using an appropriate equation.

Simple Kriging:

Ordinary Kriging:

Thus the final value is not only dependent upon the estimate in the transformed domain, but also on the associated error variance, which means the estimated value not only depends on the surrounding samples but also on the sill value of the variogram in the transformed domain.

The two reasons/advantages for log-normal kriging is:

1. Log transform reduces the variability of the sample points, thus estimating the sample points
2. By taking log normal distribution, several variables which are log-normally distributed are now normally distributed

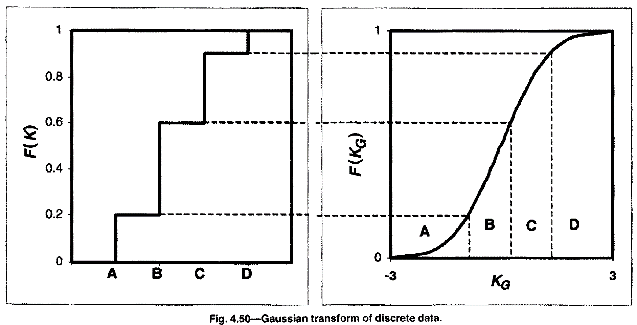
Downside of the log-normal kriging:

1. It is sensitive to the sill of the variogram for the transformed variable
2. Error variance is able to capture the configuration of the surrounding samples, but is not a true representation of the local uncertainty. Hence, Multi-Gaussian and indicator kriging are better techniques.

### Multi-Gaussian Kriging

**Steps for Multi-Gaussian Kriging:**

1. *Transform* the data, discrete or continuous into the Gaussian space, using normal score transformation
2. *Models the variogram* of the transformed variable
3. *Estimates* the value at the unsampled location using the modelled variogram
4. *Back-transforms* the estimated value in the original space.

**Transformation** of the discrete data to the Gaussian Space is done as below, by truncating the Gaussian distribution as shown. To transform, as range of continuous values are assigned for a discrete variable. To ensure proper back transform, we truncate the Gaussian distribution to a certain minimum and maximum values. (e.g. for facies A, the transformation in the Gaussian space is restricted between 3 and . Similar truncation is needed for facies D as well). When the sample data at a given location are identified as a certain facies, we know the corresponding range in the Gaussian space and any value is randomly picked within the range and assigned to the location. A more sophisticated technique can assign a value that reflects the surrounding samples.

Once the data are transformed and the **spatial relationship** is modelled, the value at the **unsampled location is estimated** by:

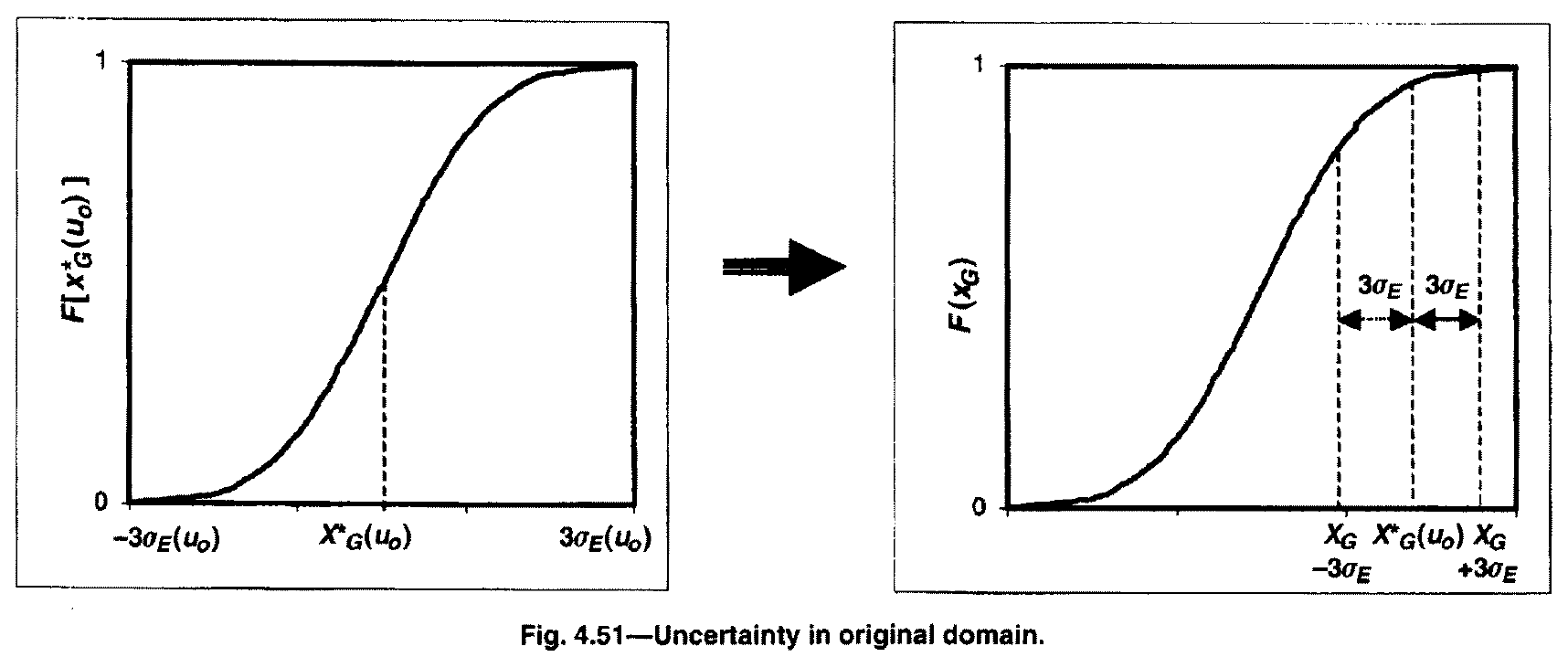
: the transformed value of the original variable

Assuming a simple kriging operation, the error variance is as below:

Once the estimated value and error variance are known, by assuming Gaussian distribution, we can completely describe the uncertainty at the unsampled location. (Transforming the data into the Gaussian domain doesn’t automatically ensure that the assumption of multivariate Gaussian distribution is valid. There are methods in the literature to check the assumption.) In most cases, the assumption is reasonable and can be used to describe the uncertainty.

**Back-transformation**: Using the estimated value and the error variance, we bracket the uncertainty in the transformed domain. Also there is a one-to-one correspondence between the transformed domain and the original data we can also define the uncertainty in the original domain.

For a discrete variable back transformation, once a value in the Gaussian space is estimated, a corresponding discrete variable can be assigned at the unsampled location by knowing the range within which it falls.



### Indicator kriging

**Characteristics of Indicator kriging:**

1. It allows the use of incomplete or soft information
2. It provides an estimate of the uncertainty at the unsampled location without any particular distribution function.
3. It minimizes variability of the sample data. Hence, a more stable variogram estimation.
4. It is helpful to understand an indicator variable in terms of the confidence in the sample value
   1. If we are 100% confident about a sampled value, it is a hard data
   2. If there is uncertainty or less than 100% confidence in the sampled value and represented by a value between 0 and 1, it is a soft data. It gives us the flexibility when assigning indicator values if the *information about a particular sample point is incomplete*.

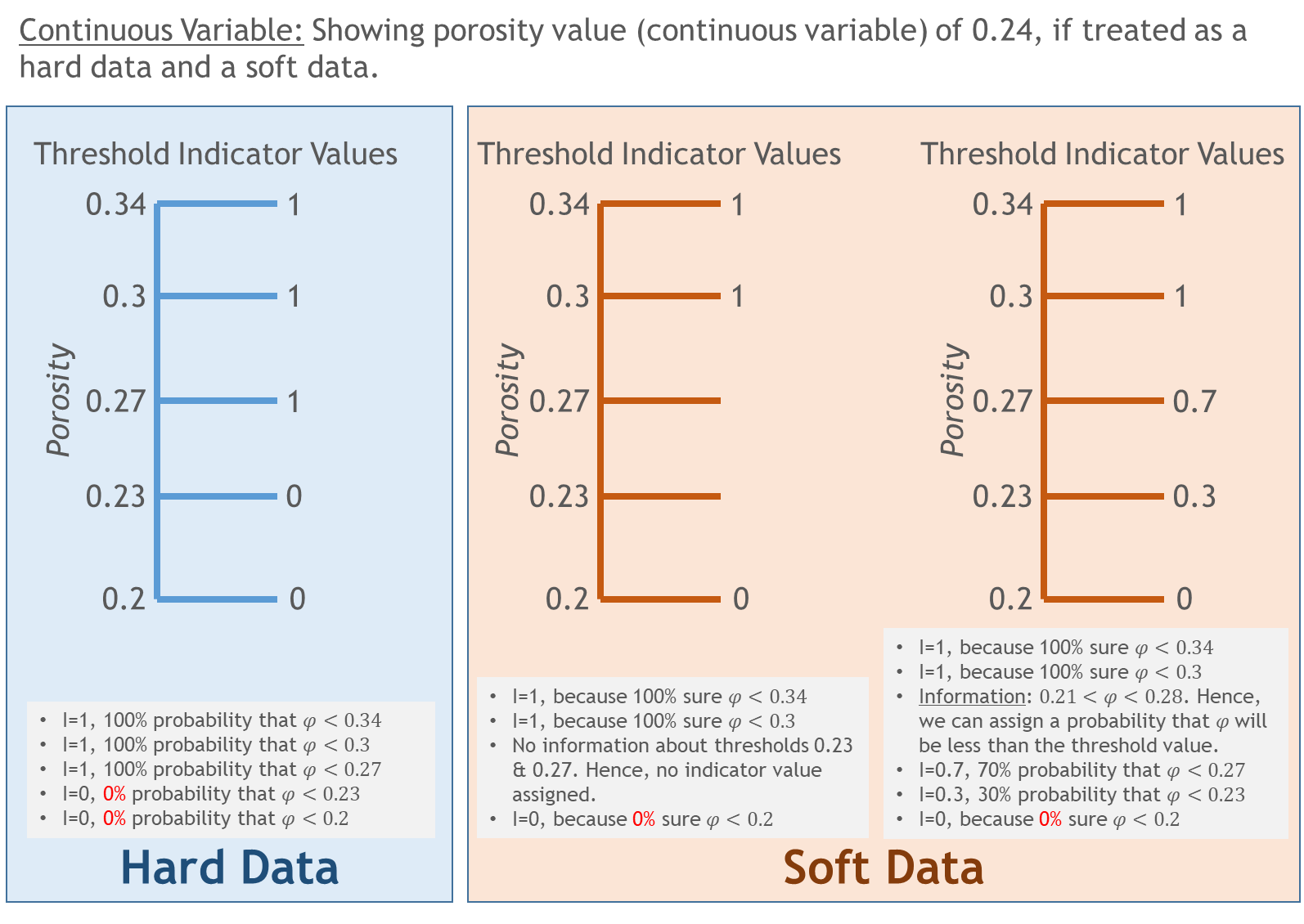
**Steps for indicator kriging:**

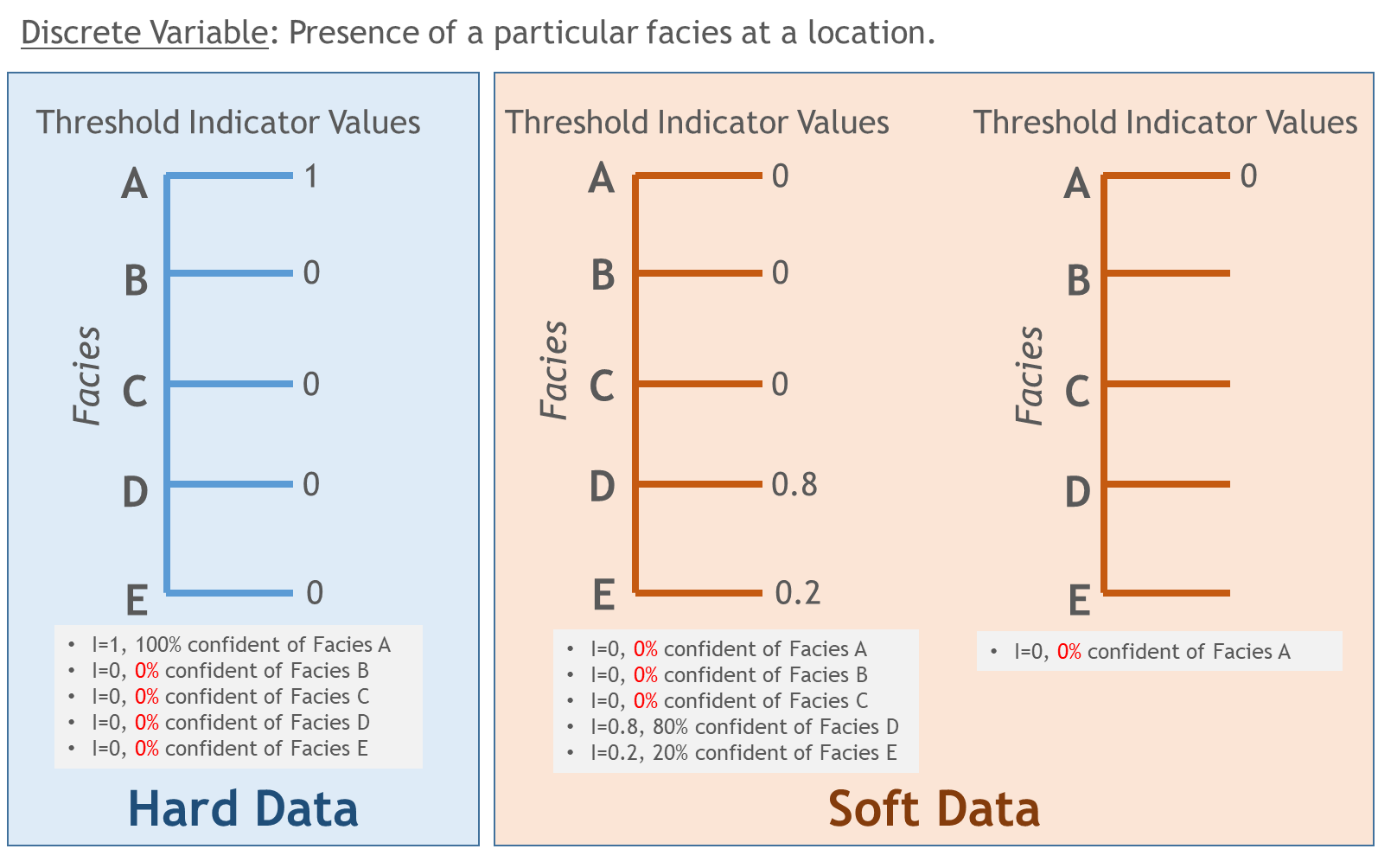
1. **Indicator transform** has to be done to either a continuous or discrete variable and the norm followed is shown in step 2. Step 1 and 2 are done simultaneously.

Indicator transform for a continuous variable:

Indicator transform for a discrete variable:

Below are **examples** shown, to better explain the differences.

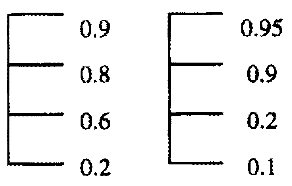


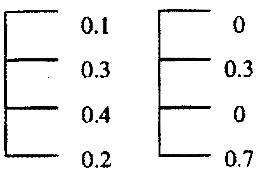


Since, indicator values represent that certain facies exists, the sum of the facies must equal to one.

1. This step involves **estimation of the indicator values** at the unsampled locations, but we need to do conventional kriging for individual thresholds.
   1. Continuous variable and simple kriging
   2. Continuous variable and ordinary kriging
   3. Discrete variable and simple kriging
   4. Discrete variable and ordinary kriging

Since in all the above cases, (also ) and , the estimate will also be .

1. After, we visit all the threshold value, we will have indicator value for each threshold values at each of the unsampled locations. We will have different results for continuous variables and discrete variables.

For continuous variables, we obtain *Cumulative Distribution Functions (CDF) at each threshold values*. E.g. at the probability that the facies lies between 2nd and 3rd thresholds is 0.8-0.6 or 0.2 in left example and 0.7 in the right one.

For discrete variables, the value against the thresholds are the *probability of occurrence of that facies*.

Adjustments has to be made to satisfy the following conditions.

1. The actual **back transformation** to the original domain is not straight forward and the common method adopted is **conditional simulation technique**.

#### Indicator Kriging to generate Permeability Thickness Map

### Probability Kriging

**Steps for probability kriging:**

1. **Transformation** of data:

Rank Index:

: ranking of the sample after the samples are arranged in ascending order.

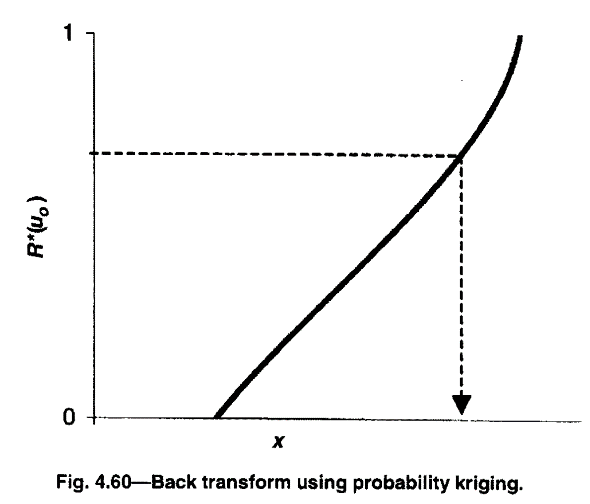
: number of sample points

Rank index falls between 0 and 1, thus reducing the variability.

1. Once rank index is obtained, **variogram** is estimated and modelled.
2. A **kriging procedure** is applied and the rank index is estimated at the unsampled location:

For simple kriging:

For ordinary kriging:

1. The estimated rank index falls between 0 and 1. Therefore, we can easily **back transform** it to the original domain as shown below.

## Estimation of uncertainty

### Parametric Estimations

### Non-parametric Estimations

### Loss Functions